

University of Saskatchewan
Department of Mathematics & Statistics

MATH 223.3 – FINAL EXAMINATION

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December 15, 2004

9 a.m.

CLOSED BOOK, NO CALCULATORS, NO ELECTRONIC DEVICES

The exam is in three parts, Part A, Part B and Part C.

Use a different Exam Booklet for each part.

Please show all of your work. The marks for each question are shown by [x]

PART A. 34 MARKS.

Answer the following questions in an Exam Booklet and label it PART A.

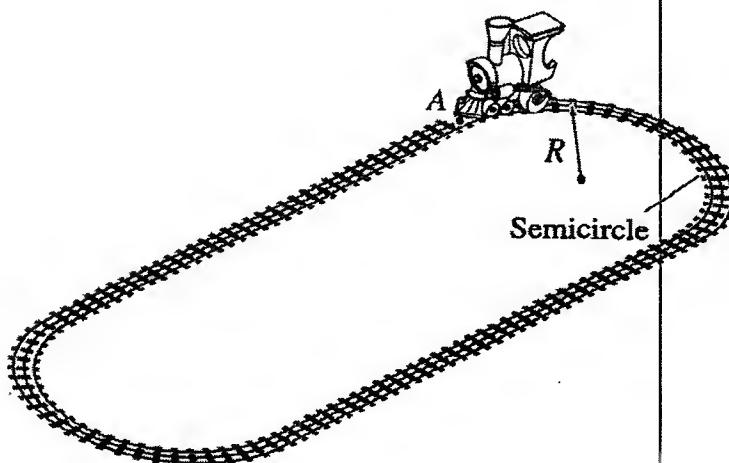
[7] 1. Find the area of the parallelogram with vertices $(1, 2, 3)$, $(4, 3, 7)$, ~~check vert~~ $(-1, 3, 6)$, and $(2, 4, 10)$. Find the equation of the plane containing these points.

[4] 2. Find the distance between the line $x = 1 - t$, $y = 2 + 3t$, $z = 4 - 2t$ and the plane $2x + 4y + 5z = 10$. ~~check not parallel~~

4 2 8
16 20
25 37
45

3. A particle has a trajectory defined by $x = t$, $y = t^2$, $z = t^2$, where t is time. Find its *velocity*, *speed* and *acceleration* at any time. What are the *normal* and *tangential* components of its *velocity* and *acceleration*? 17
17
119
170
289
25
314

[6] 4. A toy train travels *at constant speed* around an oval track which consists of semi-circular segments joined together by straight segments as shown in the figure below.



1
34
35
15
20

This problem investigates why the train tends to wobble or even derail at the point A where the semicircular segment joins the straight line segment of rail.

- (i) The position of the train at time t can be described by a vector function \mathbf{r} . Find $\mathbf{r}(t)$ which describes the position of the train before the point A and after the point A .
- (ii) Show that, with your choice of $\mathbf{r}(t)$, the speed of the train is constant.
- (iii) Show that the acceleration is discontinuous at A .

[5] 5. Find the directional derivative of $f(x, y, z) = x^3y \sin z$ at $(3, -1, -2)$ along the line $x = 3 + t, y = -1 + 4t, z = -2 + 2t$ in the direction of *decreasing* x .

[5] 6. Find $\frac{dz}{du}$ if $z = x^2yv^3$ and $x = u^3 + 2u, y = \ln(u^2 + 1)$, and $v = ue^u$.

PART B. 34 MARKS.

Answer the following questions in an Exam Booklet and label it PART B.

[5] 7. Evaluate the double integral of $(4 - y^2)^{\frac{3}{2}}$ over the region bounded by the circle $x^2 + y^2 = 4$ in the first quadrant of the xy -coordinate plane.

[5] 8. Use a double integral to find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2 - 2, y = 0$ about the line $y = -1$.

[5] 9. Use double integrals to find the centroid of the region bounded by the curves $y = e^x, y = 0, x = 0$, and $x = 1$.

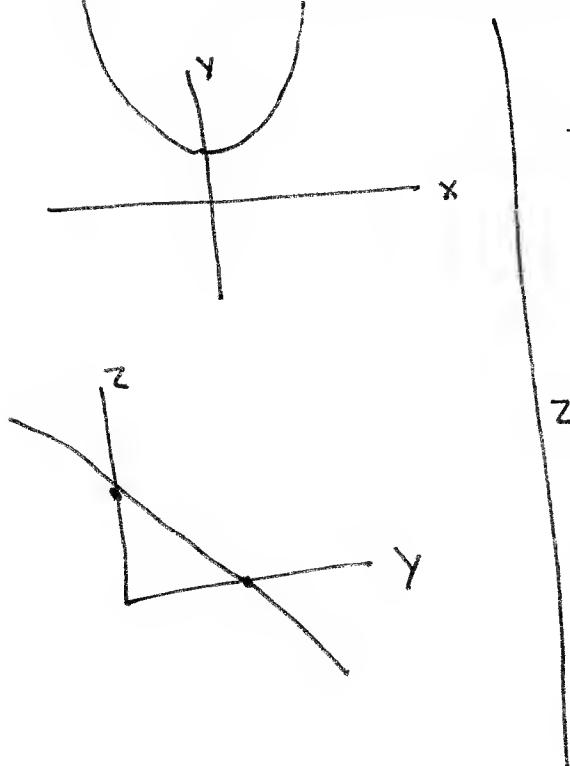
[7] 10. Evaluate a double iterated integral to find the surface area of $x^2 + y^2 + z^2 = 2$ inside the cone $z = \sqrt{x^2 + y^2}$.

[5] 11. Evaluate the triple integral of $x + y + z$ over the region V which is bounded by $z = 1 - x^2 - y^2$ and $z = 0$.

Hint:

(i) First show that the integrals of x and y are zero.
(ii) Integrate z by converting to a convenient coordinate system.

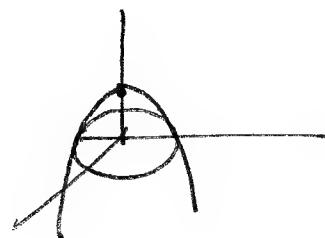
[7] 12. Find the volume bounded by the surfaces $y + z = 2, z = 0$, and $y = x^2 + 1$.



$$\begin{aligned} z: 0 &: x^2 + y^2 = 1 \\ z: 1 &: x^2 + y^2 = 0 \\ z: 2 &: x^2 + y^2 = -2 \\ z: -1 &: x^2 + y^2 = 2 \end{aligned}$$

$$z = -(x^2 + y^2 - 1)$$

$$\begin{array}{r} 15 \\ 3 \\ \hline 45 \\ -10 \\ \hline +3 \\ \hline 35 \\ \hline 3 \\ \hline 38 \\ \hline 15 \end{array}$$



PART C. 32 MARKS.

Answer the following questions in an Exam Booklet and label it PART C.

[8] 13. Verify the identities

(a) $\text{curl}(\text{grad } f) = \nabla \times (\nabla f) = \mathbf{0}$

(b) $\text{div}(\text{curl } \mathbf{F}) = \nabla \cdot (\nabla \times \mathbf{F}) = 0$

where $f(x, y, z)$ is a sufficiently differentiable scalar field and $\mathbf{F}(x, y, z) = P(x, y, z)\hat{\mathbf{i}} + Q(x, y, z)\hat{\mathbf{j}} + R(x, y, z)\hat{\mathbf{k}}$ is a sufficiently differentiable vector field

[5] 14. Evaluate the line integral $\int_C ydx + xdy + zdz$, where c is the curve $z = x^2 + y^2$, $x + y = 1$ from $(1, 0, 1)$ to $(-1, 2, 5)$.

[5] 15. Show that the line integral $\int_C 3x^2y^3dx + 3x^3y^2dy$ is independent of path. Evaluate it when c is the curve $y = e^x$ from $(0, 1)$ to $(1, e)$.

[5] 16. Use Green's theorem to evaluate the line integral

$$\oint_C (xy^2 + 2x)dx + (x^2y + y + x^2)dy,$$

$$Y^2 = \sqrt{4+x^2}$$

where c is the boundary of the region enclosed by $y^2 - x^2 = 4$, $x = 0$ and $x = 3$ and the direction of the circuit is counter clockwise.

[9] 17. Find the point(s) on the surface $z^2 = 1 + xy$ closest to the origin $(0, 0, 0)$.

Hints:

(a) Over what region in the xy -coordinate plane does the surface lie? In particular, what is the boundary of the region?

(b) It is simpler to use $(\text{distance})^2$ since it is a minimum when (distance) is a minimum.

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ y & x & z \end{vmatrix}$$

$$z_r = z_y - x_r \neq 0$$

END